

Interpreting Dark Matter Direct Detection Independently of the Local Velocity and Density Distribution

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We demonstrate precisely what particle physics information can be extracted from a single direct detection observation of dark matter while making absolutely no assumptions about the local velocity distribution and local density of dark matter. Our central conclusions follow from a very simple observation: the velocity distribution of dark matter is positive definite, $f(v) \geq 0$. We demonstrate the utility of this result in several ways. First, we show a falling deconvoluted recoil spectrum (deconvoluted of the nuclear form factor), such as from ordinary elastic scattering, can be “mocked up” by any mass of dark matter above a kinematic minimum. As an example, we show that dark matter much heavier than previously considered can explain the CoGeNT excess. Specifically, $m_\chi < m_{\text{Ge}}$ can be in just as good agreement as light dark matter, while $m_\chi > m_{\text{Ge}}$ depends on understanding the sensitivity of Xenon to dark matter at very low recoil energies, $E_R \lesssim 6$ keVnr. Second, we show that any rise in the deconvoluted recoil spectrum represents distinct particle physics information that *cannot* be faked by an arbitrary $f(v)$. As examples of resulting non-trivial particle physics, we show that inelastic dark matter and dark matter with a form factor can both yield such a rise.

I. INTRODUCTION

What particle physics information can be extracted from a detection of events at a dark matter direct detection experiment? This seemingly innocuous question is riddled with subtleties and uncertainties. The main experimental uncertainties relate to separating signal from background, and translating observed energy in a detector to nuclear recoil energy. The main astrophysical uncertainties are the local properties of dark matter for which direct detection is sensitive: the local velocity distribution and the local density.

The average density of dark matter at a galactic radius equal to the Sun is reasonably well known based on galactic kinematics (for a recent discussion, see [1]). The local density could be quite different; simulations that try to quantify the likelihood that the local density differs from the average density have been done, e.g., [2]. The simulations suggest the local dark matter density is within one to several orders of magnitude from the canonical value of 0.3 GeV/cm^3 , with more uncertainty on the lower bound than on the upper bound, depending on the likelihood one is willing to tolerate. This implies a minimum uncertainty of one to several orders of magnitude on the direct detection cross section. (As we will see, there remain several additional sources of uncertainty.)

The velocity distribution is even more subtle. *A priori* not much is known about the velocity distribution, although there has been substantial work to estimate its form using N -body simulations [3–8]. These simulations are generally excellent tracers of the *average* properties of the velocity distribution for a galaxy that is *like* the Milky Way. They do not, however, have the level of resolution needed to determine the local velocity distribution. Moreover, while most simulations do not incorporate the

feedback effects of matter, those that do find interesting “dark disk” structure [9–12]. As has been recently emphasized in [13], even isotropic spherically symmetric velocity distributions derived from equilibrium distributions of dark matter density result in departures from the Maxwellian distribution. Local effects on the density and velocity distribution by solar and gravitational capture have also been considered in [14].

Direct astronomical observations of the local stellar neighborhood can be inferred to give information on local properties of dark matter. The RAVE survey has constrained the local galactic escape speed [15]. The values lie in a fairly large range, from $460 \lesssim v_{\text{esc}} \lesssim 640 \text{ km/s}$, depending on the parameterization of the distribution of stellar velocities. Observations have also shed light on non-equilibrium local stellar motion, such as the possibility of “streams” of stars relatively nearby to the Sun. Tidal streams can arise from a disrupting satellite dwarf galaxy or star cluster. There has been evidence of streams in the solar neighborhood for some time [16–18]. More recent work with newer stellar surveys, including RAVE, SDSS, etc., suggest streams are present [19–21] (although other work does not find evidence for local streams oriented along the direction perpendicular to the galactic disk [22]). If a tidal stream of stars provides a good tracer of dark matter, this or other local structure could significantly affect dark matter direct detection measurements [10, 23–33]. Generally speaking, however, prior work has considered streams or other non-Maxwellian structure as perturbations on top of a (quasi-)Maxwellian distribution.

Our conclusion is that while the average density and average velocity distribution at the Sun’s galactic radius do seem to be determined reasonably well from both observations and N -body simulations, the *local* dark mat-

ter density and *local* dark matter velocity distribution remain highly uncertain. Breaking with the vast majority of prior literature on dark matter direct detection, we consider what particle physics properties can be unambiguously extracted while making essentially no assumptions about the density and velocity distribution of dark matter.

We approach this subject making the following simplifying assumptions:

1. One direct detection experiment reports several events that can be interpreted as dark matter scattering off nuclei.
2. Nuclear recoil events are consistent with scattering off only one type of nuclei. The scattering process off these nuclei is dominated by *either* a spin-independent or spin-dependent process, but not both.
3. No significant time variation of the rate of events is found, and no directional detection is observed.

Since the main purpose of this paper is point out what can (or cannot) be extracted from a recoil spectrum independent of local density and velocity distribution, we focus on what can be obtained from just one experiment (or, equivalently, treat several experiments' results independently). Making the second assumption is, for many experiments, not an assumption at all when detectors are made of a uniform material (e.g. Xenon100, LUX, CDMS, etc.). Even in experiments consisting of multiple nuclei with very different masses (e.g. CRESST), it would require a remarkable coincidence to have the experiment's proportions-by-mass of different nuclei arranged such that dark matter were able to scatter off several of them simultaneously with large signal over background. For spin-dependent cross sections, typically only a few isotopes dominate. Again, it would be a remarkable coincidence to have two separate scattering processes (either both spin-dependent or one spin-dependent and one spin-independent) contributing to a nuclear recoil signal in a single experiment.

The third point, that we do not consider time variation or directional dependence in rates, is a more serious omission. Indeed, time variation is itself an intriguing signal, especially given the longstanding annual modulation observed by DAMA [34]. However, we already have our work cut out for us with the comparatively mundane time-independent signal. Indeed, it is likely that convincing evidence of time variation, or directional detection, will require years of exposure. Nevertheless, some work on the effect of dark matter streams on direct detection has already been done in e.g. [23, 25]. We expect to return to this pressing issue of the purely particle physics implications of time variation and directional detection, independent of density or velocity distribution, in future work.

The upshot of these assumptions is that we consider a collection of nuclear recoil events and analyze their con-

sequences. Given the strong bounds on spin-dependent scattering from collider searches [35], we will mostly concentrate on the case of spin-independent scattering. However, our formalism and philosophy applies to both types of scattering.

We will find that some characteristics of the recoil spectrum are highly dependent on the astrophysical inputs, while others are totally independent of it. We will, of course, extract far less information than is usually presented in the literature, but the information we do extract is far more robust in the sense that it does not rely on any properties of the local dark matter halo. However, since the *only* way we can unambiguously probe the dark matter velocity distribution and abundance at the Earth's position is through direct detection, we argue that anything beyond what we claim here ultimately suffers from dependency on local astrophysical properties of dark matter.

II. KINEMATICS

We assume dark matter consists of a set of uncolored and uncharged particles χ_i with mass m_i , velocity distribution $f_i(\vec{v}_i)$, and density ρ_i . The dark matter velocities are taken to be in the Earth frame. This is a significant departure from conventional analyses. It is much more typical to take a galactic frame velocity distribution, motivated by simulations or simplifying assumptions about the galactic distribution, and boost into Earth frame. For us, since our velocity distributions $f(v)$ are specified in Earth frame, they can be used directly for direct detection scattering. However, since time-independent dark matter scattering depends only on the *magnitude* of the relative velocity, there is no way to determine the magnitude in the galactic frame outside of a kinematic range. This should be kept in mind when viewing velocity distributions later in the paper.

We will mostly consider examples with just one dark matter candidate, although the formalism we develop below will apply to an arbitrary set of WIMPs. For any given dark matter particle χ , it is useful to first consider the kinematics of the most general direct detection recoil event. We consider a collision between χ with mass m_χ , moving with velocity \vec{v} in the Earth's frame, and a nucleus N with mass m_N , that is stationary. The result of this collision, in the most general two-to-two case, will be an outgoing dark-sector particle, χ' , and an outgoing visible-sector particle, N' . We define $\delta_\chi \equiv m'_\chi - m_\chi$ and $\delta_N \equiv m'_N - m_N$, and work in the approximation that $\delta_\chi/m_\chi, \delta_N/m_N \ll 1$, i.e., the mass differences between incoming and outgoing particles are small. While generally the nucleus is not changed as a result of WIMP scattering, there are notable exceptions, such as nuclear excitation [36, 37] and rDM with neutron emission [38, 39].

In the center-of-momentum frame, where \vec{v} is the relative velocity of the two incoming particles, the incoming momentum is $\vec{p} = \mu\vec{v}$ and the outgoing momentum is \vec{p}' .

Here we use the reduced mass defined with respect to the incoming particles,

$$\mu \equiv \frac{m_\chi m_N}{m_\chi + m_N} . \quad (1)$$

The recoil energy of the collision is $E_R = q^2/2m'_N$ with

$$q^2 = p^2 + p'^2 - 2pp' \cos \theta_{\text{com}} . \quad (2)$$

The recoil of energy E_R , velocity v and $\cos \theta_{\text{lab}}$ are related by,

$$\begin{aligned} \frac{v^2}{2} \delta_\chi \frac{m_\chi}{m_{\chi'}} - v \frac{m_\chi}{m_{\chi'}} \sqrt{2m_{N'} E_R} \cos \theta_{\text{lab}} \\ - \left[E_R \left(1 + \frac{m_{N'}}{m_{\chi'}} \right) + \delta_\chi + \delta_N \right] = 0 . \end{aligned} \quad (3)$$

Define $\delta \equiv \delta_\chi + \delta_N$. If $\delta > 0$, we can safely perform an expansion in $\delta/m \ll 1$ to obtain

$$v_{\text{min}} = \frac{1}{\sqrt{2m_N E_R}} \left(\frac{m_N E_R}{\mu} + \delta \right) . \quad (4)$$

which taking $\delta_N \rightarrow 0$ is the well-known result for inelastic dark matter (iDM) [40–42]. By “safe” we mean that our upper bound on v_{min} , which is in the far non-relativistic regime, automatically implies $|\delta| \ll m_\chi, m_N$ to allow scattering to be kinematically possible.

Up to higher order terms in δ/m , we obtain an expression for the recoil energy

$$E_R^2 + 2E_R \frac{\mu}{m_N} (\delta - \mu v^2 \cos^2 \theta_{\text{lab}}) + \frac{\mu^2}{m_N^2} \delta^2 = 0 \quad (5)$$

The recoil energy is unique for a given fixed scattering relative velocity v and nucleus recoil angle θ_{lab} and can be solved by the usual quadratic formula,

$$\begin{aligned} E_R = \frac{\mu}{m_N} \left[(\mu v^2 \cos^2 \theta_{\text{lab}} - \delta) \right. \\ \left. \pm (\mu v^2 \cos^2 \theta_{\text{lab}})^{1/2} (\mu v^2 \cos^2 \theta_{\text{lab}} - 2\delta)^{1/2} \right] . \end{aligned} \quad (6)$$

This result has the well known feature that the smallest recoil energies come from maximizing $v^2 \cos^2 \theta_{\text{lab}}$, corresponding physically to head-on collisions at the highest velocities available.

III. EVENT DISTRIBUTIONS

Our basic assumptions consist of assuming the scattering process is off only one type of nuclei. We will, however, remain general with respect to the possibility of multiple WIMPs with different masses, abundances, and cross sections. One might think it requires a large coincidence to have several dark matter particles with cross sections large enough to produce events in an experiment. However, there are well known counterexamples where it can be natural to have the abundance of particles to be

independent of their mass (and thus, have several candidates of different masses with similar abundances, using for example the WIMPlless miracle [43]).

The event rate of dark matter scattering [44], differential in E_R , is determined by

$$\frac{dR}{dE_R} = \sum_i \frac{N_T \rho_{\chi_i}}{m_{\chi_i}} \int_{v_{i,\text{min}}}^{v_{\text{max}}} d^3 \vec{v}_i f_i(\vec{v}_i(t)) \frac{d\sigma_i |\vec{v}_i|}{dE_R} , \quad (7)$$

where the sum is over different species of WIMPs, $m_N \simeq Am_p$ is the nucleus mass with m_p the proton mass and A the atomic number. The recoil energy depends on the kinematics of the collision, as described above. Given our assumption of no significant time variation in the rate, $f(\vec{v}_i(t)) \rightarrow f(\vec{v}_i)$, and thus we are effectively neglecting the Earth’s motion around the Sun. This is a reasonable approximation so long we are probing velocities larger than Earth’s velocity in the Sun’s frame, i.e., $v_{\text{max}} \gtrsim 30$ km/s. Typically the maximum speed is taken to be $v_{\text{max}} = v_{\text{earth}} + v_{\text{esc}}$, the galactic escape velocity boosted into the Earth frame. However, v_{max} is ultimately determined by the (unknown) details of the dark matter velocity distribution in Earth frame.

Given our assumption of no direction dependent signal, we can carry out the angular integral in Eq. (7), reducing it to a one dimensional integral where we introduce the quantity¹ $f_1(v) = \int d\Omega f(\vec{v})$. The differential rate becomes

$$\begin{aligned} \frac{dR}{dE_R} = \sum_i \frac{N_T \rho_{\chi_i} m_N}{\mu_i^2 m_{\chi_i}} F_N^2(E_R) \\ \times \int_{v_{i,\text{min}}}^{v_{\text{max}}} dv_i v_i f_{i1}(v_i) \bar{\sigma}_i(v_i, E_R) , \end{aligned} \quad (8)$$

where we have written

$$\frac{d\sigma_i}{dE_R} = F_N^2(E_R) \frac{m_N}{\mu_i v_i^2} \bar{\sigma}_i(v_i, E_R) \quad (9)$$

in terms of the nuclear form factor $F_N^2(E_R)$. There are several possible forms for the scattering cross section $\bar{\sigma}_i(v, E_R)$, depending on the interaction,

$$\bar{\sigma}_i(v, E_R) = \begin{cases} \sigma_{i0} \\ \sigma_{i0} F_{\chi_i}^2(E_R) \\ \sigma_{i0}(v) F_{\chi_i}^2(E_R) \\ \sigma_{i0}(v, E_R) \end{cases} . \quad (10)$$

The different forms for $\bar{\sigma}$ correspond to functional forms of known dark matter scattering that contain velocity and/or recoil energy dependence. The first possibility, a constant independent of v and E_R is the well-known isotropic (s-wave) cross section that results at lowest order in the non-relativistic expansion from many dark matter models.

¹ The velocity distribution is normalized such that $\int d^3 v f(v) = 1$.

The second possibility contains a dark matter form factor F_{χ_i} (following the standard normalization convention $F_{\chi_i}(E_R = 0) = 1$) and commonly occurs in models of composite dark matter [45–47]. Our formalism will handle the factorizable forms, i.e., the first three of Eq. (10), which incorporates the vast bulk of what has been considered in the literature. We will not, however, consider the cross sections that contain completely arbitrary nonfactorizable velocity and recoil energy dependence [c.f., the most general form written on the fourth line of Eq. (10)].

We now turn to the question of what can be inferred from a signal in direct detection experiments using (8) without making any assumptions about f_1 or the dark matter scattering cross section σ_0 . We will however, make an assumption about the maximum dark matter speed, v_{\max} , and we will demonstrate how the derived dark matter properties depend on this assumption.

IV. DECONVOLUTED SCATTERING RATE

Since the scattering rate (8) in any given direct detection experiment is proportional to the nuclear form factor, we first factor it out. This leads to a definition of a new quantity, \mathcal{R} , that we call the “deconvoluted scat-

tering rate” – deconvoluted of the nuclear form factor,

$$\mathcal{R} \equiv \frac{1}{F_N^2(E_R)} \frac{dR}{dE_R} = \sum_i \mathcal{N}_i m_N \int_{v_{i,\min}}^{v_{\max}} dv_i v_i f_{i1}(v_i) \bar{\sigma}_i(v_i, E_R). \quad (11)$$

Some overall factors have been buried into a normalization factor, $\mathcal{N}_i = N_T \rho_{\chi_i} / (\mu_i^2 m_{\chi_i})$. While there are important uncertainties in the determination of dark matter nuclear form factors from nuclear data [48], this is not our concern. Errors on the deconvoluted scattering rate ought to take into account nuclear form factor uncertainties.

Next, taking a derivative with respect to E_R we find

$$\frac{d\mathcal{R}}{dE_R} = \sum_i \mathcal{N}_i m_N \left(\int_{v_{i,\min}}^{v_{\max}} dv_i v_i f_{i1}(v_i) \frac{d\bar{\sigma}_i(v_i, E_R)}{dE_R} - v_{i,\min} \frac{dv_{i,\min}}{dE_R} f_{i1}(v_{i,\min}) \bar{\sigma}(v_{i,\min}, E_R) \right). \quad (12)$$

For arbitrary $2 \rightarrow 2$ kinematics (elastic or inelastic), we can replace

$$v_{i,\min} \frac{dv_{i,\min}}{dE_R} = \frac{m_N^2 E_R^2 - \mu_i^2 \delta_i^2}{4m_N \mu_i^2 E_R^2}. \quad (13)$$

This is as far as we can go with a general signal from an ensemble of WIMPs with arbitrary cross sections.

For a single WIMP with a factorizable cross section, Eq. (11) can be used to solve for $f_1(v)$ (see also [49–52]):

$$f_1(v_{\min}(E_R)) = -\frac{4\mu^2 E_R^2}{m_N^2 E_R^2 - \mu^2 \delta^2} \frac{1}{\mathcal{N} \sigma_0(v_{\min}(E_R)) F_\chi^2(E_R)} \left(\frac{d\mathcal{R}}{dE_R} - \mathcal{R} \frac{1}{F_\chi^2(E_R)} \frac{dF_\chi^2(E_R)}{dE_R} \right). \quad (14)$$

This result allows us to gain information on the velocity distribution of dark matter evaluated at the *minimum* velocity to scatter for a given recoil energy E_R . With scattering data over the range $E_R^{\min} < E_R < E_R^{\max}$, we obtain information on the velocity distribution $f(v)$ over a range of v : $v_{\min}(E_R^{\min}) < v < v_{\min}(E_R^{\max})$.

For an ensemble of WIMPs, χ_i , without dark matter form factors, the inversion result can be written as

$$\frac{d\mathcal{R}}{dE_R} = \sum_i w_i(v, E_R) f_{i1}(v), \quad (15)$$

where the velocity distributions of the WIMPs are “weighted” by the factors

$$w_i(v, E_R) = -\frac{1}{4} \left(\frac{m_N^2}{\mu_i^2} - \frac{\delta_i^2}{E_R^2} \right) \mathcal{N}_i \sigma_{i0}(v) \quad (16)$$

For an ensemble of WIMPs *with* form factors, no simple closed form can be written.

V. f -CONDITION

There is valuable information that can be extracted from Eqs. (14) and (15). We know the velocity distribution of dark matter must be positive for all v ,

$$f(v) \geq 0, \quad (17)$$

which we call the “ f -condition”. Using this condition, the right-hand side of Eq. (14) must be positive. Similarly the f -condition also places constraints on the terms appearing in Eq. (15).

Consider the case of single WIMP with standard elastic scattering without a dark matter form factor, $\delta = 0$ and $F_\chi^2(E_R) = 1$. From Eq. (14) we conclude that the deconvoluted scattering rate is always a decreasing function of E_R .

A more striking consequence is reached if a rising deconvoluted scattering rate is ever observed. Should there be a range of data where the deconvoluted scattering rate

satisfies $d\mathcal{R}/dE_R > 0$, this would signal the presence of either an inelastic threshold for scattering or a dark matter form factor (or both). This can be seen most easily by classifying which terms in Eq. (14) can be positive or negative.

This general characteristic of the shape of the nuclear recoil distribution can thus be used to provide an experimental signal of non-standard dark matter interactions, *independent* of the local velocity distribution and local density of dark matter. This observation is one of main conclusions of our paper.

Eq. (14) illustrates how the introduction of non-trivial particle physics can allow for structure in the recoil spectrum while still satisfying Eq. (17), through either an inelastic cross section or one with a non-trivial dark matter form factor. One smoking gun for inelastic dark matter is a rise and fall of the (deconvoluted) dark matter scattering rate. Yet, Eq. (14) makes clear that a dark matter form factor whose functional form has positive powers of E_R over a range of E_R within the experimental sensitivity also can lead to a rise and fall of the deconvoluted scattering rate. Since no direct detection experiment can probe $F_\chi^2(E_R = 0)$, the functional form of a dark matter form factor is essentially unconstrained.² Indeed, Ref. [53, 54] demonstrated that specific models of what they called “form factor dark matter” can reproduce a rising deconvoluted scattering rate, $d\mathcal{R}/dE_R > 0$, like inelastic dark matter. It is interesting to consider simple functional forms for the form factor $F_\chi^2(E_R)$, and whether they can (or cannot) fake inelastic dark matter given an arbitrary velocity distribution. We carry out this study in Sec. VID.

The f -condition also allows us to see that there is a complete degeneracy between $\sigma(v)$ and $f_1(v)$. No one experiment can separate a velocity-dependent dark matter scattering cross section from astrophysical structure in the velocity distribution. To the extent that multiple experiments do not have the same $\sigma(v)$ for different nuclei, it should be possible to break the degeneracy through multiple observations.

It is instructive to consider several possible forms of $f_1(v)$ and how they map into distributions of energies. As a simple starting point, consider a velocity distribution that can be approximated by a delta function,

$$f_1(v) = \delta(v - v_0). \quad (18)$$

This could correspond physically to a local density of dark matter that is contained within a stream moving with speed v_0 in the Earth’s frame, or a scattering cross section with a resonant feature [39]. From Eq. (11), the deconvoluted spectrum will be a constant provided $v_{min} \leq v_0$ and zero otherwise. In terms of E_R , the rate

is constant provided E_R is below a certain threshold determined by v_0 , and zero above it. This result is shown graphically for elastic scattering off xenon in the top panels of Figure 1.

One can build a discretized version of any $f_1(v)$ by combining a suitable number of delta functions. Every delta function contributes to the energies below its threshold E_R , and zero above it. In the case of elastic scattering, the result is a deconvoluted distribution which falls off at larger energies. Two other simple cases, $f_1(v) = \text{constant}$ for some range of v and $f_1(v)$ falling linearly in v are also shown in Fig. 1.

Several further comments are in order. If we imagine a spectrum of several delta functions, the lowest velocity contributions may result in recoil energies *below* an experiment’s detection threshold, and therefore obviously cannot be observed. Given that the weights of these delta functions are also undetermined, this means that the normalization of the velocity distribution cannot be experimentally determined.

Experiments may also be limited in sensitivity at for some recoil energy $E_R > E_R^{\text{exp,max}}$. Contributions to the velocity distribution at very high velocities that contribute to the recoil spectrum in this range would also not be fully probed by an experiment. This is another source of uncertainty regarding the normalization of the velocity distribution.

The upshot of this is that there is both *astrophysics* as well as fundamental *experimental sensitivity* limitations in interpreting the overall normalization of a given recoil spectrum as arising from a given cross section. This limitation is fundamental – for an arbitrarily large fraction of dark matter moving slowly enough in Earth frame, no experiment will be sensitive to such nuclear recoils, and thus, no experiment can measure a cross section independently of astrophysics. However the results we find here are based on the shape of the recoil spectrum and not the normalization, and thus can be determined independent of astrophysics.

VI. CASE STUDIES

We now consider several examples of what can, and cannot, be determined from a nuclear recoil spectrum of a single dark matter detection experiment. Given the discussion above, in all cases the normalization of $f_1(v)$ (and thus the cross section) is completely arbitrary.

A. Elastic Dark Matter

For the case of elastic dark matter (eDM), defined as standard elastic scattering ($\delta = 0$) with no dark matter form factor ($F_\chi^2(E_R) = 1$), we can determine a lower bound on m_χ provided one has knowledge of the maximum possible velocity, v_{max} . From Eq. (5), we can solve for μ as a function of E_R and the other parameters. For

² We are not aware of a way to obtain $F_\chi^2(E_R) \propto E_R^n$ for $n < -2$ from a microscopic theory. However, since this does not lead to $d\mathcal{R}/dE_R > 0$, this limitation is not of much concern.

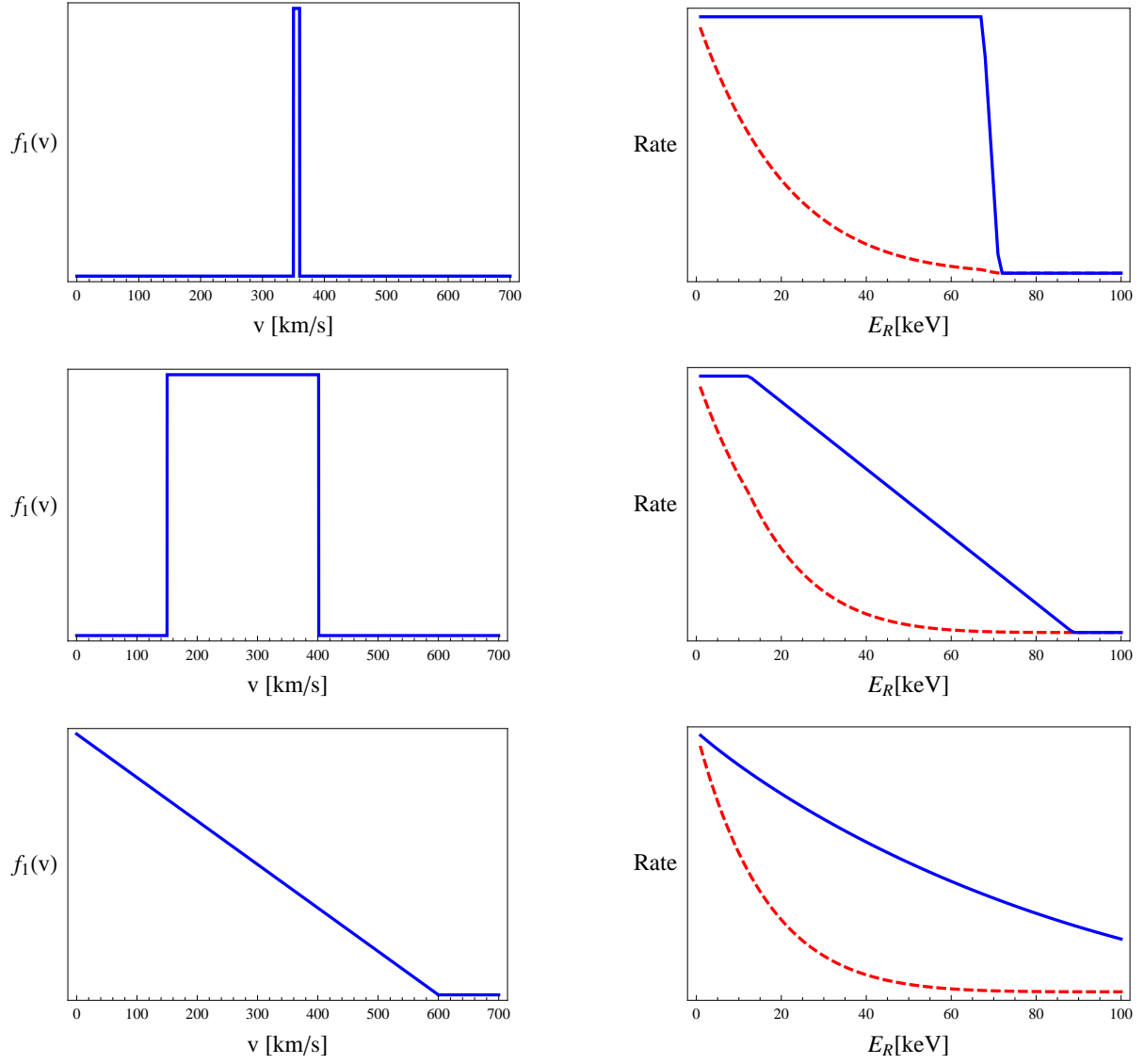


FIG. 1. Some examples of the relationship between velocity distribution (LH plots) and observed recoil spectrum, dR/dE_R , (red dashed in RH plots) and deconvoluted spectrum, \mathcal{R} , (blue solid in RH plots). The dark matter mass is taken to be 100 GeV with elastic scattering off xenon.

elastic scattering, the result is particularly simple,

$$\mu_{\min} = \sqrt{\frac{m_N E_R}{2v_{\max}^2}}, \quad (19)$$

demonstrating that the strongest lower bound on the dark matter mass comes from the highest recoil energy events at the maximum dark matter velocity (in Earth frame). We illustrate this bound in Fig. 2, by showing the bound on m_{χ}^{\min} as a function of v_{\max} for four possible values of the maximum recoil energy from a distribution of events where dark matter scatters off xenon. In the next section, we will see that the analogous constraints on m_{χ}^{\min} for inelastic dark matter depends on the inelastic threshold.

The deconvoluted scattering rate, Eq. (14), takes on the simple form in eDM,

$$f_1(v) = -\frac{4E_R}{m_N^2 \mathcal{N} \tilde{\sigma}_0(v)} \frac{d\mathcal{R}}{dE_R}. \quad (20)$$

The positivity of $f(v)$ (and $\tilde{\sigma}_0(v)$) means that for elastic scattering the spectrum of recoil events *must* be a monotonically decreasing function of energy. If this is not observed, we can immediately rule out elastic scattering, completely independently of any assumptions about how it couples to nuclei or how it is distributed in our galaxy.

We now discuss what can be determined if indeed a falling spectrum is observed. As a surrogate for experimental data, and to demonstrate our technique, we generate pseudo-data for a 100 GeV WIMP elastically scat-

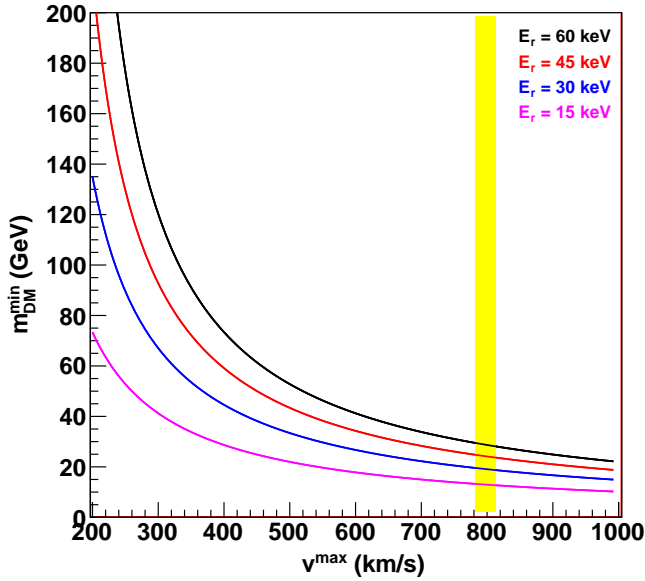


FIG. 2. The kinematic bound on the dark matter mass, assuming elastic scattering off xenon. Different curves are plotted for different assumptions about what events are assumed to be the highest energy nuclear recoils from dark matter. The yellow band corresponds to the typically largest relative velocity arising from summing $v_{esc} + v_{earth} \lesssim 800$ km/s.

tering off xenon, assuming the standard Maxwellian distribution for the dark matter velocity in galactic frame. Specifically, our input contains a distribution with characteristic speed, $v_0 = 220$ km/s (in galactic frame), with an escape speed of $v_{esc} = 500$ km/s (in galactic frame). Since we do not consider time-dependent signals, we take the Earth’s velocity to be a constant, $v_{earth} = 230$ km/s (in galactic frame). We consider the lower threshold of our pseudo-experiment to be recoil energies of 5 keV and the upper threshold we take to be 80 keV (the latter happens to be below the first zero of the nuclear form factor of xenon). These specifications are a reasonable approximation of the capabilities of the Xenon100 experiment [55]. We ignore detector effects such as energy resolution and efficiency and assume that these are sufficiently well known that the recoil energy distribution of Eq. (7) can be determined from the data.

Next, we invert this “data” using Eq. (20) and assume the dark matter is scattering elastically. For simplicity, we use the same scattering cross section used to generate the data. This only affects the normalization of $f_1(v)$ and does not alter any of our arguments. In Fig. 3 we show the derived velocity distributions $f_1(v)$ for various choices of dark matter mass. As expected, for the correct choice of dark matter mass the derived velocity distribution agrees with that used to generate the data. However, since data is only taken over a finite range of recoil energies the velocity distribution is only known over a finite range of velocities, corresponding to the v_{min} associated

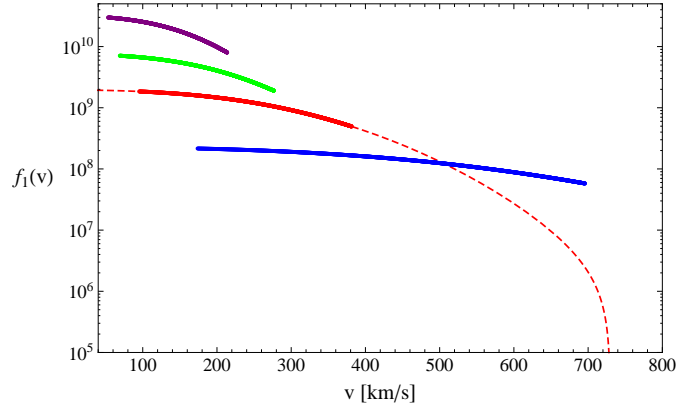


FIG. 3. Velocity distributions derived from pseudo-data, with $5 \text{ keV} \leq E_R \leq 80 \text{ keV}$, generated for a 100 GeV WIMP elastically scattering of xenon. The data is generated assuming a Maxwellian distribution for the dark matter, dashed red line. For $m_\chi = 40, 100, 500$ GeV (thick line segments, from bottom to top) the derived velocity distributions, $f_1(v)$, are shown.

with the E_R , Eq. (4).

As was discussed in Sec. II it is possible to place a lower bound on the dark matter mass by assuming a maximum speed for dark matter in our halo, the highest energy recoil events, taken to be 80 keV in the above, then determine a minimum mass through Eq. (4). However, as can be seen from Fig. 3 no such upper bound can be made. For all assumed masses used in Fig. 3 the resulting velocity distributions appear *a priori* to be perfectly reasonable: $f_1(v)$ is positive and finite. Without further model-dependent assumptions, or additional experimental results (either from another experiment or from raising the upper threshold at the first experiment), all dark matter masses, $m_\chi \geq m_\chi^{min}$, give a reasonable fit to data. Using Eqs. (4) and (20) in the limit of very heavy dark matter, the derived velocity distribution becomes independent of the dark matter mass and only depends on the target.

B. Elastic Dark Matter: CoGeNT

A further interesting example of elastic dark matter is the recent observation of an excess of low energy recoil events by CoGeNT [56]. The CoGeNT collaboration demonstrated that this is consistent with light dark matter, 7-10 GeV, where the mass was determined by taking the standard Maxwellian velocity distribution.

Following our procedure above, we have taken a fit to the low recoil energy excess at CoGeNT [57] and reverse-engineered the distribution to determine the needed velocity distribution for (much) larger dark matter masses in Fig. 4. It is easy to understand the shift in the velocity distributions. For light dark matter, $m_\chi < m_{\text{Ge}}$, the maximum recoil energy is $E_R^{\text{max}} \simeq m_\chi^2 v_{\text{max}}^2 / m_{\text{Ge}}$. Once

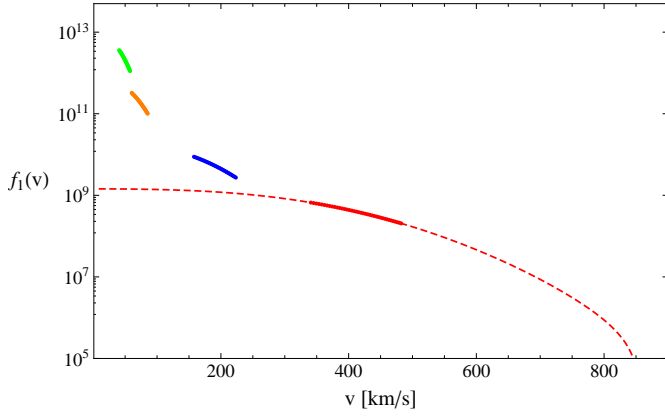


FIG. 4. Velocity distributions derived from the CoGeNT data. The dashed red line is the ordinary Maxwellian velocity distribution for an elastically scattering dark matter particle that fits the CoGeNT excess with a mass of 8 GeV. The derived velocity distributions, $f_1(v)$, are shown for $m_\chi = 8, 20, 100, 500$ GeV (thick line segments, from bottom to top).

the dark matter mass is larger than about m_{Ge} , the maximum recoil energy asymptotes to $E_R^{\text{max}} \simeq m_{\text{Ge}} v_{\text{max}}^2$. The shift in v_{max} from $m_\chi \simeq 7$ GeV to $m_\chi \gg m_{\text{Ge}}$ is thus a factor of $\simeq 10$. Indeed, from Fig. 4 we see that the largest minimum velocity shifts from about 500 km/s to about 60 km/s as the mass is increased from 8 GeV to 500 GeV.

By itself, the CoGeNT data can thus be fit by *any* dark matter candidate above the kinematic minimum, which for $v_{\text{max}} \sim 800$ km/s is about 6 GeV. The conclusion that CoGeNT implies “light dark matter” is thus not warranted if the local dark matter density and velocity distributions can be freely adjusted.

The obvious objection to the large dark matter, small velocity distribution dark matter interpretation of the CoGeNT data is that other direct detection experiments should be sensitive to larger masses, and potentially rule it out. We can test this assertion by translating the CoGeNT observed energies into nuclear recoil energies (following [58]), and then compute the predicted spectrum at an experiment using xenon. This assumes, of course, that whatever scattering process is occurring in germanium also occurs in xenon.

In Fig. 5 we show the predicted recoil spectrum at a xenon experiment, for each of the velocity distributions shown in Fig. 4. In each case, the prediction is large numbers of recoil events below 6 keVnr. This may or may not be allowed by existing data from Xenon10 [59] and Xenon100 [55]. The principle difficulty is determining the sensitivity of these experiments to very low recoil energies, specifically the conversion factor $L_{\text{eff}}(E_R)$. There has been a spirited discussion on this point [60–66]. Clearly, masses much heavier than considered by CoGeNT are allowed, while arbitrarily large masses (represented by 500 GeV) may be constrained by this existing

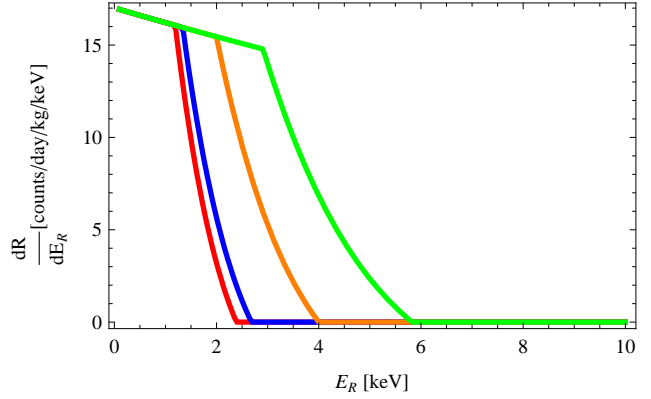


FIG. 5. Predicted recoil spectra at a Xenon experiment for the velocity distributions that fit the CoGeNT excess shown in Fig. 4. The lines correspond to $m_\chi = 8, 20, 100, 500$ from left to right. Notice that the normalization corresponds to thousands of events at Xenon10 or Xenon100.

or future xenon data. In any case, our central conclusion is that qualitatively larger dark matter masses can fit the CoGeNT data if the velocity distribution is adjusted as we illustrated above.

C. Inelastic Dark Matter

Due to the splitting of inelastic dark matter, v_{min} is not monotonic in E_R , which means that the highest energy recoil events may not give the strongest bound on the dark matter parameters. The expression for v_{min} , Eq. (4), has a minimum, and correspondingly the deconvoluted recoil spectrum, \mathcal{R} , has a peak at

$$E_R^{\text{peak}} = \frac{m_\chi}{m_\chi + m_N} \delta, \quad (21)$$

the observed spectrum, dR/dE_R , typically has a peak at a lower energy due to the nuclear form factor. Whether the highest or lowest energy bins of an experiment place the strongest constraints on m_χ and δ depends on where this peak falls.

If E_R^{peak} is large enough there may even be an observable gap between an experiment’s lower threshold and the first recoil events – this bump-like spectrum aids in fitting the energy spectrum of the DAMA modulation data. On the other hand, if E_R^{peak} is below the lower threshold it is not possible to distinguish iDM from conventional elastic dark matter. In Fig. 6 we illustrate these points for the case of a xenon experiment which records data over the range $5 \text{ keV} \leq E_R \leq 80 \text{ keV}$.

Unlike eDM, where the recoil spectrum must be monotonically decreasing, the mass splitting of iDM allows for the spectrum to be a rising function for $E_R < E_R^{\text{peak}}$. This cannot be mimicked by elastic scattering without

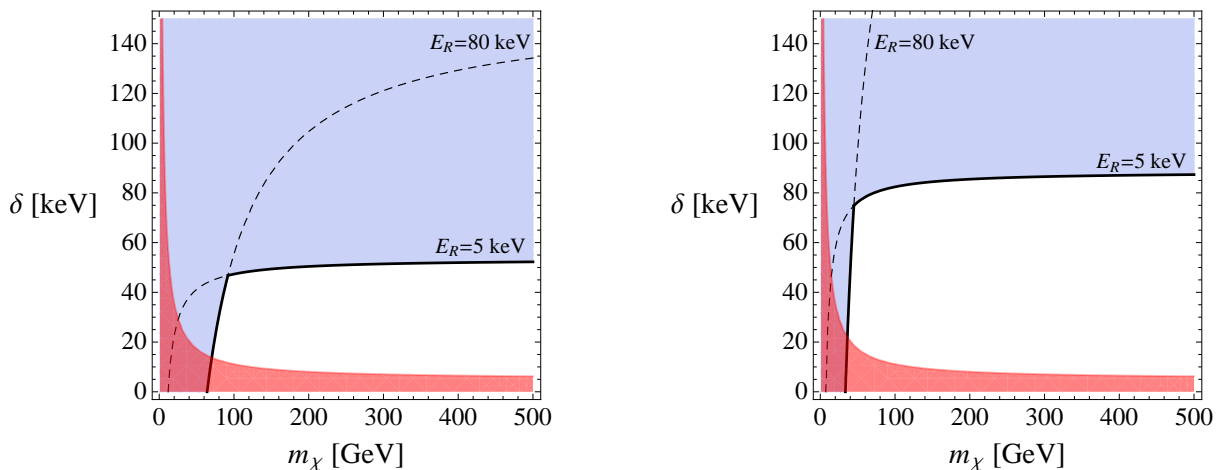


FIG. 6. The astrophysics independent allowed region of iDM parameter space (unshaded region) for a recoil spectrum with events, in a xenon detector assuming a maximal speed for dark matter of 500 km/s (LH plot) and 800 km/s (RH plot) in the Earth’s frame. In both plots the two curves assume events observed with 5 and 80 keV. The lower (red shaded) region is model independent and is the region in which the peak, Eq. (21), in the deconvoluted recoil spectrum lies below 5 keV and iDM cannot be distinguished from eDM. With reliable knowledge of $f_1(v)$ this region extends further upwards.

a dark matter form factor, no matter what (physically allowed) form the velocity distribution takes. Furthermore, at E_R^{peak} the denominator in (14) has a zero but since $f_1(v)$ must be finite the recoil spectrum must have a maximum at this same energy so the numerator goes to zero as well. Thus the observation of a maximum in the recoil spectrum determines E_R^{peak} and reduces the 3 dimensional iDM parameter space by one dimension, relating m_χ and δ , independent of knowledge of the dark matter velocity distribution.

D. Form Factor versus Inelastic

Although the spectrum of iDM cannot be faked by simple elastically scattering DM, regardless of what physical velocity distribution is used, it may be faked by elastically scattering dark matter that has a non-trivial form factor. Due to the unique kinematics of iDM, induced by the splitting of the states, a deconvoluted iDM spectrum has a gap in the low energy spectrum with no recoil events. The size of this gap, and thus the position of the first events is

$$E_R^{\text{gap}} = \frac{\mu^2}{m_N} \left(v_{\text{max}}^2 - \frac{\delta}{\mu} - \sqrt{\left(v_{\text{max}}^2 - \frac{\delta}{\mu} \right)^2 - \frac{\delta^2}{\mu^2}} \right). \quad (22)$$

This can be obtained directly from Eq. (7) by calculating the lowest recoil energy for the highest velocity ($v = v_{\text{max}}$) for a head-on collision ($\cos \theta_{\text{lab}} = 1$). The spectrum then increases to E_R^{peak} , Eq. (21), before again decreasing. Near the peak, the spectrum is well fit by a power law but away from the peak the spectrum becomes more exponential in nature. This increase in \mathcal{R} between

E_R^{gap} and E_R^{peak} would require an unphysical $f_1(v) < 0$ if the dark matter has no form factor. However, it is possible that a non-trivial $F_\chi(E_R)$ could “overpower” this increase and allow iDM spectra to be fit by eDM with a form factor, albeit for a non-Maxwellian velocity distribution. We demonstrate this below for the simple example of a form factor that is a single power in E_R , $F_\chi \sim E_R^n$, for a sufficiently high n the resulting $f_1(v)$ will be sensible.

Although the increase in the recoil spectrum can be accommodated by elastically scattering dark matter with a non-trivial form factor, the existence of a gap with iDM cannot be faked with a simple power-law form factor. This is because the threshold to up-scatter results in a step function turn on in the spectrum, and no such step function results from a simple polynomial form for $F_\chi^2(E_R)$.

To illustrate the ability of iDM to be faked by form factor dark matter. We consider the simple case of $F_\chi = \left(\frac{q}{q_0}\right)^{2n}$ with the normalization $q_0 = 10^{-3}\mu$ and we take $n \in [-1, 10]$ and investigate whether there is a physically sensible velocity distribution that allows form factor dark matter (with the same WIMP mass) to fake iDM with $m_\chi = 100$ GeV, with a Maxwellian velocity distribution, for various splittings δ . This region in which iDM can indeed be faked is shown as the shaded region in Figure 7 and an example velocity distribution that achieves this is shown in Fig. 8.

VII. DISCUSSION

We have shown that certain particle physics properties can be determined from the nuclear recoil spectrum

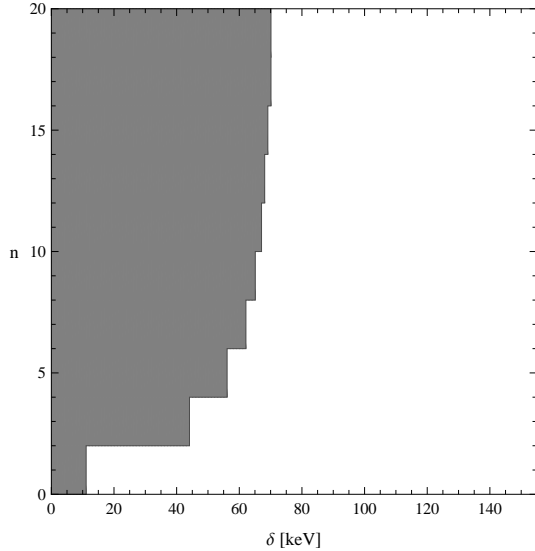


FIG. 7. In the shaded region elastically scattering dark matter with a form factor $F_\chi \propto q^{2n}$ can fake iDM with splitting δ . The dark matter mass was taken to be 100 GeV, scattering off xenon, and the iDM spectrum was assumed to come from a Maxwellian distribution.

of a dark matter direct detection experiment independent of the local astrophysical density and velocity distribution. These properties are most easily uncovered from the deconvoluted scattering rate, that we call \mathcal{R} . We advocate that experiments present their recoil spectrum data in terms of the deconvoluted spectrum, from which the shape of \mathcal{R} as a function of E_R can be read off independently of the nuclear form factor.

The kinematics of scattering leads to a lower bound on dark matter mass, given a maximum speed for the WIMP in the Earth’s frame, independent of the details of the dark matter velocity distribution. We have also shown that an upper bound on the mass of dark matter, however, *cannot* be determined independently of the density and velocity distributions of dark matter. This was demonstrated for both “fake” Xenon data, but also for the CoGeNT excess. Perhaps the most striking consequence is that we find the CoGeNT excess can be fit by dark matter with masses much heavier than has been generally considered. Much of this parameter space, up to $m_\chi \simeq m_{\text{Ge}}$, is no more strongly constrained than light dark matter, given an appropriate choice of velocity distribution. For $m_\chi > m_{\text{Ge}}$, the CoGeNT data may or may not be allowed by existing Xenon10 and Xenon100 data, which ultimately depend on the details of $L_{\text{eff}}(E_R)$ and Q_y .

Our analysis comparing different masses, as well as comparing and contrasting inelastic dark matter with form factor dark matter, was done without any experimental errors. It would be interesting to carry out similar analyses with experimental errors. For example, the

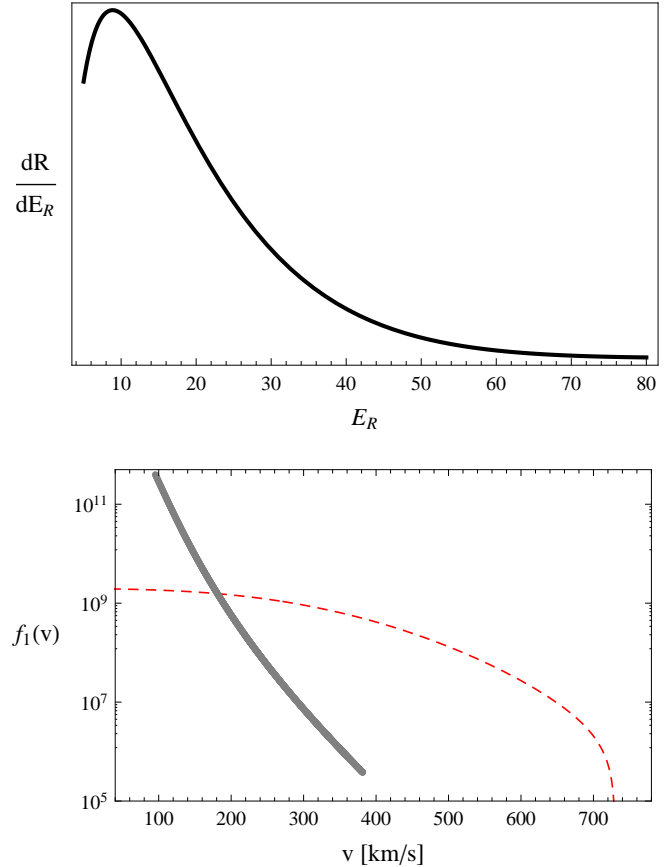


FIG. 8. Top: a recoil spectrum, at a xenon based detector, for iDM with $m_\chi = 100$ GeV and $\delta = 35$ keV within the range $5 \leq E_R/\text{keV} \leq 80$. Bottom: the iDM Maxwellian velocity distribution is shown as the red dashed curve. The solid gray curve corresponds to the velocity distribution that exactly matches the recoil spectrum shown in the top figure, but with dark matter, of the same mass, scattering elastically with a form factor $F_\chi \propto q^4$.

so-called “gap” in inelastic dark matter may not be resolvable due to experimental errors, so that it may be reproduced by a smooth dark matter form factor.

Breaking from the vast majority of past literature, we do not impose any requirements of the velocity distribution of dark matter, except for assuming there is an upper bound (that might or might not be equated to $v_{\text{earth}} + v_{\text{esc}}$ in Earth’s frame). In fact, our velocity distributions were taken to be in Earth’s frame, and thus to compare with most other literature one would have to boost into galactic frame.

Interestingly, however, there are potential physical realizations of some of the more unusual velocity distributions found in this paper. In particular, distributions with velocities that are smaller than the Earth’s velocity in the galactic frame ($\simeq 230$ km/s) could be interpreted in at least two ways. One is dark matter that streams

in a direction parallel to the motion of the Sun around the galaxy. This could be a local stream, or could be a “dark disk”. Another interpretation is that the lower velocities arise from dark matter that is weakly bound to the Earth/Sun/Jupiter system. In both cases, relative velocities in the tens, rather than hundreds, of km/s are physically realizable.

There are still many questions and many interesting avenues for future exploration of astrophysics-independent implications of dark matter. Time-dependence in the scattering rate is perhaps the most important, given the ongoing interest in the annual modulation observed by DAMA. In the future, directional dependence will also provide an excellent way to determine the nature of the dark matter velocity distribution we find ourselves in. Understanding the particle physics implications of these observations is left to future work. With information about dark matter from other sources, such as mass determinations from the LHC, the approach presented here could allow the interpretation of direct detection results as a direct measurement of the local as-

trophysical properties of dark matter. Finally, we have not considered the implications of multiple nuclear recoil spectra from different experiments. This too is an very interesting and timely subject that we hope to see work on very soon [67].

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